សង្ខិត្តន័យ—Abstract

អត្ថបទទី៦៖ការវាយតម្លៃចំណេះដឹងគណិតវិទ្យាក្នុងការបង្រៀនខ្លឹមសារប្រភាគរបស់គរុសិស្សកម្រិតបឋមសិក្សានៅកម្ពុជា

សង្ខិត្តន័យ

សិស្សកម្រិតបឋមសិក្សាមួយចំនួននៅក្នុងប្រទេសកម្ពុជា នៅមានកង្វះខាតលើចំណេះដឹងមូលដ្ឋាននៃមុខវិជ្ជាគណិតវិទ្យាជាពិសេសមេរៀនប្រភាគ ហើយលទ្ធផលសិក្សាខ្សោយរបស់សិស្សគឺទាក់ទងទៅនឹងការបង្រៀនរបស់គ្រូ។ គ្រូបង្រៀននៅកម្រិតបឋមសិក្សានៅមានចំណេះដឹងលើខ្លឹមសារ និងគរុកោសល្យ (PCK) នៅមានកម្រិតនៅឡើយដូចជាវិធីសាស្ត្រក្នុងការបង្រៀន និងការប្រើប្រាស់រូបតំណាង។ ដូចនេះការសិក្សានេះមានគោលបំណងការវាយតម្លៃលើចំណេះដឹងរបស់គរុសិស្សកម្រិតបឋមសិក្សា (PPSTs) អំពីចំណេះដឹងគណិតវិទ្យាក្នុងការបង្រៀន (MKT) លើខ្លឹមសារប្រភាគ។ ការសិក្សានេះបានប្រើប្រាស់វិធីសាស្ត្រសិក្សាស្រាវជ្រាវចម្រុះ ដោយមានការចូលរួមពីគរុសិស្សកម្រិតបឋមសិក្សាមកពីគ្រឹះស្ថានបណ្តុះបណ្តាលគ្រូចំនួនពីរ។ គរុសិស្សចំនួន២០៦នាក់ បានបំពេញកម្រងសំណួរចំនួន១៨សំណួរ លើចំណេះដឹងគណិតវិទ្យាក្នុងការបង្រៀនដែលផ្តោតលើបញ្ញត្តិរងទាំងប្រាំនៃប្រភាគ ហើយគរុសិស្សចំនួន៤៥នាក់ ត្រូវបានជ្រើសរើសដើម្បីចូលរួមសម្ភាសន៍។ ការសិក្សានេះបានរកឃើញថាគរុសិស្សទទួលបានលទ្ធផលខុសៗគ្នាទៅតាមបញ្ញត្តិរងរបស់ប្រភាគ។ ក្នុងចំណោមបញ្ញត្តិរងទាំងនោះ គរុសិស្សទទួលបានលទ្ធផលទាបជាងគេទៅលើបញ្ញត្តិរងដែលប្រភាគជាផ្នែកនៃវត្ថុទាំងមូល (part-whole)។ គរុសិស្សបានចាត់ទុកថាប្រភាគដែលជាផ្នែកនៃវត្ថុទាំងមូលជាប្រភាគដែលបែងចែកជាទំហំប៉ុនគ្នា និងមានការលំបាកក្នុងការបង្កើតរូបតំណាងផ្សេងគ្នាៗរបស់ប្រភាគ។ លើសពីនេះទៅទៀត គរុសិស្សមានចំណេះដឹងមិនគ្រប់គ្រាន់ផងដែរលើផ្នែកខ្លឹមសារឯកទេសគណិតវិទ្យាដោយពួកគេមិនអាចពន្យល់បានពីហេតុផលនៃនិតិវិធីក្នុងការដោះស្រាយលំហាត់របស់ពួកគេ។ កម្រិតទាបនៃចំណេះដឹងផ្នែកខ្លឹមសារឯកទេសគណិតវិទ្យាធ្វើឱ្យចំណេះដឹងផ្នែកខ្លឹមសារ និងគរុកោសល្យនៅមានកម្រិតផងដែរ។ គរុសិស្សមានការលំបាកក្នុងការពន្យល់ពីគំនិតភាន់ច្រឡំរបស់សិស្ស និងលំបាកក្នុងការប្រើប្រាស់រូបតំណាងប្រភាគ។ ទោះបីជាយ៉ាងណាក៏ដោយ ជាទូទៅពួកគេបានយល់ដឹងពីការប្រើប្រាស់បញ្ញត្តិរងដែលប្រភាគជាផ្នែកនៃវត្ថុទាំងមូលមកបង្កើតជាឧទាហរណ៍ក្នុងការបង្រៀនខ្លឹមសារប្រភាគ។

ពាក្យគន្លឹះ៖ គរុសិស្សកម្រិតបឋមសិក្សា ចំណេះដឹងគណិតវិទ្យាក្នុងការបង្រៀន ចំណេះដឹងខ្លឹមសារ និងគរុកោសល្យ ចំណេះដឹងលើខ្លឹមសារមេរៀន បញ្ញត្តិរងទាំងប្រាំនៃប្រភាគ

Examining Cambodian Pre-service Primary School Teachers' Mathematical Knowledge for Teaching (MKT) on Fractions

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Abstract

Primary school students in Cambodia lack basic skills in mathematics, especially in fractions—the poor performance related to teachers' instruction. Many primary school teachers still have inadequate Pedagogical Content Knowledge (PCK) regarding instructional strategies and representations. Therefore, this study aims to examine Cambodian pre-service primary school teachers' (PPSTs) Mathematical Knowledge for Teaching (MKT) on fractions. This study employed an explanatory sequential mixed-method design involving pre-service primary school teachers from two teacher education institutions. 18 items of MKT, which focused on the five sub-constructs of fractions, were administered to 206 PPSTs, and 45 PPSTs were selected for interviews. This study found that PPSTs' performance varied by sub-construct of fractions. Among the five sub-constructs, the part-whole sub-construct received a poor result. PPSTs considered part-whole as an equal shape and struggled to provide different types of representation. Moreover, PPSTs also had inadequate Specialized Content Knowledge (SCK); they could not explain the reason behind the procedure of their answers. The limitation of SCK causes a limited PCK. They were difficult to explain students' misconceptions and utilize the representations of fractions. However, they were familiar to utilized part-whole sub-construct for posing examples in teaching fractions.

Keywords: Pre-service primary school teachers; Mathematical knowledge for teaching; Pedagogical content knowledge; Subject matter knowledge; Five sub-constructs of fractions

1. Introduction

In Cambodia, education is a priority area that aims to develop human capital for national competitiveness and economic growth and to meet the economic goal of the Association of Southeast Asian Nations (ASEAN) to transform the country from a lower-middle-income to an upper-middle-income country by 2030 (Royal Government of Cambodia [RGC], 2014). The road map of Cambodia's Sustainable Development Goal (SDG) 2030 sets up the strategies for meeting the education goal of improving teachers' quality, which is stated as the main objective in the Education Policy. To achieve this objective, Cambodian teachers have been provided with opportunities to attend the appropriate training to develop their professional competence. They have also been motivated and supported to develop sufficient academic content knowledge and pedagogical skills (Ministry of Education Youth and Sports [MoEYS], 2019). To improve teachers' quality, MoEYS has made an effort to attract and motivate recent high school graduates who are strongly committed and have the required competencies to be involved in teacher education. Moreover, the curriculum of the pre-service teacher education program has been upgraded from 12+2 to 12+4 programs. Currently, two Teacher Education Colleges in Cambodia implement the 12+4 program for pre-service teacher training. This is the first step for the expansion of the teacher education system (MoEYS, 2015; JICA, 2017).

Pre-service teacher education plays a significant role in improving teachers' quality for a more extended period (Prigent, 2016). Studies by Ma (1999) and Wright (2008) in mathematics education also showed that the pre-service teacher education programs need to emphasize conceptual understanding of mathematics and subject matter knowledge (SMK) for future teachers. The researchers also acknowledged the primary concern of connecting the pre-service teachers' mathematics competence with the teaching and learning in school mathematics (Ma, 1999; Wright, 2008). In 2012, The study of Teacher Education and Development Study in Mathematics (TEDS-M) provided evidence for understanding how pre-service mathematics teachers gain content knowledge and pedagogical content knowledge during their study in the teacher education colleges. The TEDS-M study also noted that the study of pre-service teachers' knowledge regarding content knowledge and pedagogical content knowledge (PCK) could be a crucial indicator for measuring the success of the teacher education program (TEDS-M, 2012).

A recent study by Van, Moa, and Cnudde (2018) suggested that improving the teacher preparation program with a focus on subject matter knowledge (SMK) and pedagogical content knowledge (PCK) of future teachers is in critical demand in Cambodia In the educational system, the teacher quality is considered as a core element as it is significantly related to students' performance (Phin, 2014; Sem & Hem, 2016; Song, 2012). PCK is at the core of teachers' professional knowledge and it is recognized as a type of knowledge that helps transfer subject matter knowledge to the students (Shulman, 1986; Hill, Rowan, & Ball, 2005; Kilic, 2011). Moreover, PCK has emerged as a crucial topic that has been significantly linked to students' achievement (Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005; Ngo, 2013).

The World Bank report (2015) on Improving Teacher Quality in Cambodia revealed that teacher trainers, practicing teachers, and pre-service teachers have inadequate pedagogical content knowledge (PCK) in diagnosing and remedying students' misconceptions (Tandon & Fukao, 2015). Moreover, a study by Van et al. (2018) found that although teacher trainers in Provincial Teacher Training Centers (PTTCs) had improved their content knowledge (CK) and PCK after the intervention, they still struggled with PCK, implying the ineffectiveness of instructional strategies and representations.

Furthermore, the results of the Early Grade Mathematics Assessment (EGMA) indicated that first-grade students lagged behind in building the basic mathematics competencies for primary school students. They could read the given numbers, yet their conceptual knowledge of number sense was insufficient (MoEYS, 2016). According to the Cambodian National Assessment of

Mathematics conducted by the Education Quality Assurance Department in 2013, sixth-grade students lacked basic skills in the mathematics domain (MoEYS, 2015). They had better procedural knowledge but insufficient conceptual comprehension (MoEYS, 2015, 2016). For instance, although the Number domain was the main content at the primary level, students attained a poor result in solving fraction problems. The percentage of correct answers to questions on operations of fractions and mixed numbers was on average 38.1 percent. Surprisingly, the percentage of correct answers to the addition and subtraction of fractions was merely 13.5 percent. The report revealed that students' poor performance was related to teachers' instructions. It noted that teachers put more emphasis on theories and abstract concepts of mathematics and demonstrated the lesson of the content with insufficient teaching aids (MoEYS, 2015). Therefore, it was suggested that it is crucial to enhance early grade fundamental skills with respect to the fraction concept (MoEYS, 2016).

In the number domain, fractions has been recognized as a complicated topic for learning and teaching at the primary level because of the diverse meanings of fractions, especially in the early grades (Van Steenbrugge et al., 2014). Research has shown that students tended to lack conceptual understanding of fractions; thus, they learned fractions without understanding the meaning of fractions (Hansen, Jordan, & Rodrigues, 2017; Siegler & Lortie-Forgues, 2015). Meanwhile, pre-service teachers had a limited understanding of the fractions concept and the explanation of the rationale behind the procedure of doing fractions problems (Kilic, 2015; Newton, 2008; Wright, 2008).

Thus, to understand what Cambodian pre-service primary school teachers' (PPSTs) have learned in their training college regarding Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK), this study is conducted. It aims to examine PPSTs' Mathematical Knowledge for Teaching (MKT) on fractions. The specific research question is: What is Cambodian pre-service primary school teachers' Mathematical Knowledge for Teaching (MKT) on fractions?

2. Literature review

2.1 The concept of Mathematical Knowledge for Teaching (MKT)

Shulman (1986) introduced the notion of pedagogical content knowledge (PCK), referring to the knowledge of how to teach and make a particular subject comprehensible for students. The notion of PCK is linked between content knowledge and practice (Kilic, 2011; Hill, Rowan, & Ball, 2005;Shulman, 1986). Shulman and his colleagues began the study with prospective teachers on "*knowledge that grows in the minds of teachers, which focus on the content*." They distinguished three categories of content knowledge, such as subject matter content knowledge, pedagogical content knowledge, and curricular knowledge (Shulman, 1986).

In 2008, Ball and her colleagues argued that the Shulman framework presented an inadequate relationship between theoretical and empirical perspectives Ball, Thames, & Phelps, 2008). The term PCK was widely used, but it is still underspecified. The insufficient definition and practical foundation limited the benefit of PCK. To improve the foundation of PCK more comprehensively, the connection between knowledge and practice need to be investigated (Ball, Thames, & Phelps, 2008). Ball et al. (2008) project interconnected teachers' knowledge, teaching, and students' learning (Chua, 2018). To develop their conceptual framework, Ball et al., (2008) reviewed Shulman's ideas as the foundation. They utilized the bottom-up approach, which started by exploring the knowledge that is required in teaching mathematics to students. The main question was emphasized on the knowledge employed in teaching rather than the teachers, which focused on "*what teachers need to know the content, determine what else teachers need to know about mathematics, and how and where teachers might use such mathematical knowledge in practice*" (Ball et al., 2008, p.394-395). The framework of Mathematical Knowledge for Teaching (MKT) was then developed, which referred to the mathematical knowledge that is needed to carry out teaching work. The study about teachers' mathematical knowledge for teaching provided a positive association between teachers' knowledge and students' learning outcomes (Ball et al., 2008; Depaepe et al., 2013).

The MKT model consists of two components: Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK), each of which has three categories of knowledge. Subject Matter Knowledge (SMK) is split into common content knowledge (CCK), which refers to the knowledge used in teaching, and it is used in many other tasks and professions that also employed mathematics (simply calculating an answer, correctly solving mathematics problems) ; for example, write fractions correctly based on the representation and ability to calculate the fractions operations. Specialized Content Knowledge (SCK) refers to the knowledge that allows the teachers to engage in specific teaching tasks, including how to represent mathematics ideas accurately. Moreover, teachers can provide mathematics explanations for common rules or procedures as well as examine and understand unusual solutions or problems. For example, teachers can explain the meaning of fractions by using the representations and understand the reason behind the calculation. The third component, Horizontal Content Knowledge (HCK), refers to teachers' awareness of how mathematical topics are related to the curriculum span included in the curriculum such as the ability to consider which topic relates to Simplify Fractions. Moreover, The component of Pedagogical Content Knowledge (PCK), is divided into the knowledge of content and students (KCS), which refers to teachers' knowledge of students' conceptions and misconceptions and possible difficulties about a particular topic and the ability to diagnose misconceptions. For instance, teachers can identify students' misconceptions about learning fractions concepts. Knowledge of Content and Curriculum (KCC) refers to knowledge of learning goals for different grades and knowledge of instructional materials. For example, teachers can understand students' prior knowledge and content based on the curriculum. Knowledge of Content and Teaching (KCT) refers to teachers' ability to choose appropriate tasks, examples, or representations for a particular group of students and the ability to eliminate students' misconceptions. For instance, teachers can utilize a variety of examples and representations for teaching fractions concepts. According to Depaepe et al. (2013), the concept of MKT has three clear merits.

> First, "MKT is the result of the empirical study on the knowledge that teachers need for and apply in teaching mathematics". Second, "MKT furthers the operationalization of Shulman's concept through the development of a valid measure of teachers' mathematical knowledge for teaching". Third, "the concept of MKT provides empirical

evidence for a positive relationship between teachers' PCK and student learning outcomes" (Depaepe et al., 2013).

Based on the existing literature, if was found that mathematics teachers and pre-service teachers had inadequate pedagogical content knowledge. They had difficulties in diagnosing and determining students' misconceptions about fractions. Moreover, their subject matter knowledge regarding the specialized content knowledge also insufficient (Ball et al., 2008; Depaepe et al., 2015; Newton, 2008; Purevdorj & BABA, 2009; Turnuklu & Yesildere, 2007; Van et al., 2018; Wright, 2008;).

2.2 Primary school students and pre-service teachers' knowledge of fractions

Fractions were invented when the whole number cannot fully describe a mathematical situation. There are various ways to represent fractions useful to develop children's understanding of fractions concepts utilizing the visual representation of the linear model, area model, and set model (Musser, Peterson, & Burger, 2003, pp. 215-223). Kieren (1993) distinguished five sub-constructs of fractions for developing students' understanding as follows:

- Part-whole: A number of equal parts of a unit out of the total number of equal parts into which the unit is divided.
- Measure: A distance, a position in a number line that fractions as an abstract number. Fractions as the quantity, which helps the student to see fractions as the number with magnitude.
- Quotient: Fractions are regarded as the result of the division, which helps students understand the relative size of fractions.
- Operator: Comprises the application of a function to a number, an object, or a set.
- Ratio: Fractions is a comparison between two quantities, it can be a part-part or part-whole relationship based on the context. (Kieren, 1993)

The different meanings of fractions occurred in different situations and contexts (Van Steenbrugge et al., 2014). It is necessary to teach students to understand the various and interrelated meanings of fractions and the integration of these facets. Fraction knowledge is useful in everyday skills. Notably, knowledge of the fractions concept is extremely important because it linked to fractions achievements, algebra, and other advanced mathematics in the higher grades (Jordan et al., 2013; Mohsin & Baba, 2007).

However, in the early grades of the primary level, students tend to have some difficulties in learning fractions. Primary school students face hardship in understanding the notation of fractions and misread fractions. Their prior knowledge of whole numbers also has influenced and confused their learning of fractions (Hansen et al., 2017; Lortie-Forgues & Siegler, 2015; Nguyen et al., 2017). Lamon (2012) noted that shifting from the whole number into fractions creates the variety and complexity of the situation. Moreover, fractions were understood as a difficult topic in teaching and learning because of its various meanings (Van Steenbrugge et al., 2014). The various meanings of the fractions symbol were concerning. In 2013, the

Cambodian National Assessment of Mathematics found that the 6th-grade students could not solve fractions problems properly. The majority of the students were confused with the numerator and denominator in solving addition and subtraction of fractions (MoEYS, 2015). To teach the operations of fractions, we need to understand the fractions' sense and the broad range of the phenomena that form the meaning underlying the fractions symbol (Lamon, 2012, p. 32).

Previous studies have shown that pre-service teachers had inadequate conceptual knowledge; on the contrary, they performed better in procedural knowledge of fractions, especially in fraction multiplication and division (Ball, 1990; Kilic, 2015; Van Steenbrugge et al., 2014). They lacked the ability to explain the rationale of a procedure or the underlying conceptual meaning of fractions. Moreover, it was found that pre-service teachers have limited understanding of the particular meaning of fractions; for instance, they were more familiar with the part-whole sub-construct than the other sub-constructs. A challenge of the PCK they encountered was to identify students' misconceptions and instructional representation (Kolar et al., 2018; Lee, Son, & Arabeyyat, 2015; Reeder & Utley, 2017; Van Steenbrugge et al., 2014; Wright, 2008).

A quantitative study by Van Steenbrugge et al. (2014) investigated pre-service teachers' knowledge of the five sub-constructs of fractions. They utilized the fraction test items to measure pre-service teachers' fractions knowledge. The study found that pre-service teachers had better procedural knowledge performance; however, their specialized content knowledge received a low result. Among the five sub-constructs of fractions, the participants accomplished a better result in doing part-whole and ratio sub-construct problems but were deficient in the measure sub-construct. The ratio sub-construct was strongly related to the two proportions and part-whole sub-construct (Van Steenbrugge et al., 2014).

3. The development of the fractions concept in Cambodian mathematics textbooks

In the Cambodian context, textbooks are considered an important resource material for teaching and learning. Teachers utilize textbooks as the main source for classroom activities and homework that emphasize the computation tasks (Song, 2015). Chan (2015) raised some common issues related to mathematics textbooks at the primary level. For example, mathematics textbooks often contained some errors, which novice teachers cannot recognize, and therefore they ended up teaching incorrect mathematics to students. The content in mathematics textbooks employed less modern psychological and pedagogical approaches. The content and its applications also provided little evidence related to real-life activities (Chan, 2015).

Fractions are recognized as an important topic that is related to daily life. Understanding fractions and its meaning encourage students to understand the operations of fractions (Jordan et al., 2013; Kilic, 2015; Mohsin & Baba, 2007). However, the multifaceted constructs of fractions are challenging for students who need to learn to acquire a deep understanding of the nature of fractions. In the case of primary school students in Cambodia, it was found that they could attain better procedural knowledge but had insufficient conceptual knowledge. In addition, they struggled in doing fractions tasks in the national examinations (MoEYS, 2015, 2016). This study analyzed the Cambodian mathematics textbooks at the primary level focusing on various meanings of fractions. The five sub-constructs of fractions and their representation were analyzed based on Lamon's (2012) framework, which represents Part-whole, Measure, Quotient, Operator, and Ratio. The textbook analysis aimed to understand the development of the fraction concept regarding the multi-aspects of fractions and integration of these aspects in each grade and identify how fractions were introduced regarding the representations of fractions concept in each example. The textbook analysis resulted that each grade at the primary school focused on the part-whole sub-construct of fractions. In the Cambodian textbook, the part-whole sub-construct is commonly used to develop the understanding of fractions concept. The operator sub-construct is utilized, but, in this part, just focused on the multiplication of fractions and whole numbers. The other sub-constructs of fractions were not presented in the textbook. The table below summarizes the analysis results, from grade 1 through grade 6, that focused on examples of teaching and learning fractions. Sixty examples were posed as examples to introduce fractions.

Table 1
Result of textbook analysis on fractions sub-construct and representations

	Sub-constructs of Fractions					Representation of fractions			
	P-W	Meas	Quot	Oper	Ratio	Area	line	set	number
N (60)	40	4	0	1	0	36	0	4	39
%	66.66	6.66	0	1.66	0	60	0	6.66	65

Mathematics textbooks tended to provide examples of the part-whole sub-construct (66.66%) and utilize the area model (60%) to introduce the fractions. Moreover, 65 percent of the examples have written in numbers, which focuses on the demonstration of fraction calculations. There were limited examples used area of fractions as the part-whole sub-construct; it leads teachers and students with a constrain in developing their knowledge or understanding of fractions.

4. Methodology

4.1. Research design

This study employed an explanatory sequential mixed-method design to obtain reliable and valid data to answer the research questions (Creswell & Clark, 2011). The explanatory sequential mixed-method design is a two-phase method that uses qualitative data to explain the result of the quantitative data. In this study, the participants for the qualitative study were purposively selected based on the quantitative result. However, the qualitative data in this study were not only used to explain the quantitative result, but it was utilized to answer the research question regarding the pre-service primary school teacher' pedagogical content knowledge (PCK) and subject matter knowledge (SMK).

4.2. Research setting and participants

The study was conducted in Phnom Penh Teacher Education College (PPTEC) and Svay Rieng Provincial Teacher Training Center (SVPTTC). PPTEC is located in Phnom Penh city, the capital of Cambodia. Currently, PPTEC is reforming pre-service training programs from 12+2 to 12+4 programs (JICA, 2017; MoEYS, 2015). SVPTTC is not on the list of reforms for Teacher Education College (JICA, 2017; MoEYS, 2015). At present, SVPTTC is the institution that trains primary school teachers for the 12+2 program with a small number of pre-service teachers who participated in a survey designed for this study. In the survey, all second-year pre-service teachers (a total of 206 pre-service primary school teachers), who have attended the mathematics method courses about fractions based on the written curriculum in PPTEC and SVPPTC, were encouraged to join the study.

Among the 206 teachers, there are 123 pre-service teachers (78 female) who are being trained in the 12+4 program, and 41 pre-service teachers (26 female) in the 12+2 program in PPTEC. There are 42 pre-service teachers (29 female) who are attending the 12+2 training program in SVPTTC. All these 206 teachers participated in this study.

In the next phase of the mixed-method design, 45 (26 female) pre-service primary school teachers (PPSTs) were purposively selected for interviews through clustered and systematic sampling methods. They were selected based on their performance of the MKT test. After getting the MKT test result, the PPSTs score was grouped based on their training institutions and training programs. The results showed that there were three groups, from the lowest to the highest of the MKT test result. The researcher then selected the participants in every interval based on the formula of a systematic sampling method $f=\frac{N}{Sn}$ (that f =frequency interval; N= the total number of the sample; Sn= the required number in the sample) (Cohen, Manion, & Morrison, 2007).

4.3. Research instruments

The MKT questionnaire contained two parts. The first part required participants' demographic information about gender, age, math grade, and education qualification. The test items in the second part were developed to measure PPSTs' mathematical knowledge for teaching focusing on the five sub-constructs of fractions, which was adapted from previous studies (Depaepe et al., 2015; Kilic, 2015; Kolar, Čadež, & Vula, 2018; Peruvdorj, 2009; Tanaka, 2019; Van Strenbrugge et al., 2014;). The researcher checked the correspondence of the test items with the Cambodian mathematics curriculum and textbook at the primary school level and teacher education programs. As a result of the pilot survey, 18 items of MKT have been retained, in which 17 items were SMK (15 CCK items and 2 SCK items) and 1 item which intended to measure PCK focused on KCT. The Cronbach Alpha estimated the reliability of the questionnaire at .610, which was considered as moderate reliability. Meanwhile, in the interview sessions, the questions were utilized to measure PPSTs' knowledge about students' difficulties in solving this item (KCS) and how to utilize this item as the representation and appropriate examples for teaching and learning fractions (KCT) in primary school.

4.4. Data collection procedure

A pilot survey was conducted to validate the questionnaire items. There were 24 pre-service teachers who involved in the pilot survey. After completing the pilot survey questionnaire, they were asked to check the statements and questions' spelling and meaning. For the MKT test, there were 34 items on subject matter knowledge (SMK) and pedagogical content knowledge (PCK) of fractions. As a result of the pilot survey, 16 items were removed, and most of the items related to SCK and PCK required more written explanation. However, most of the participants did not complete these questions because it took time to write and explain in handwriting. The researcher removed the items which were not completed by more than 60 percent of the participants. After removing the items, the MKT test was revised based on comments from the participants and senior mathematics educators. As a result, 18 items were retained in the main survey, and the removed items were examined again in the interviews.

In the survey, participants were given enough time and a brief explanation of how to respond to the questionnaires was provided. The MKT test result was used to select the participants who completed the questionnaire for the interviews. The interviews were conducted approximately two weeks after the survey. The interviews were conducted face-to-face using a semi-structured interview protocol, which was developed to identify PPSTs' SCK and PCK. The interviews were audio recorded with handwriting notes and memos.

4.5. Data analysis procedure

Before starting the data analysis, data cleaning was conducted to screen the missing data of the quantitative analysis. The PPSTs' responses from the MKT questionnaire were analyzed based on the rubric that considered the correctness of the responses. The quantitative data was analyzed using SPSS software. This study employed descriptive statistics that focus on the frequency and the central tendency of the data. Chi-square and one-way ANOVA were employed to examine the significant difference of the data. Moreover, the normality of the MKT result was analyzed using a Shapiro-Wilk test, a widely used test, that has a strong ability to detect the normality of the data.

The data derived from the interviews were analyzed based on the steps of analyzing the qualitative data suggested by Creswell (2014). The first step started with organizing and preparing the data for analysis that included transcribing the interview data and sorting the data into different types based on the information sources. Then, the researcher read all the data and code the data to generate major themes. Finally, the data were interpreted (Creswell, 2014). After transcribing and coding the interview data, each interviewee's data was utilized to verify the result of the MKT questionnaire. Lastly, after confirming both types of data, the researcher generated key themes reflected in the components of the MKT model.

5. Results

5.1. General performance of PPSTs' subject matter knowledge on fractions

In the SMK part of the MKT questionnaire, as shown in Table 2, the highest expected score was 17. The descriptive statistics indicated that the mean score (M) of the PPSTs was 7.56, and standard deviation (SD) was 2.72 with 95% confidence interval for the mean (lower bound = 7.19, upper bound = 7.94). These results revealed that pre-service teachers' general performance is quite low. The result of the Shapiro-Wilk test (statistics .987 and the statistically significant values $p = .056$) revealed that the p value is not significant ($p > .05$) (See Table 2). This result showed that the PPSTs' performance is normally distributed.

Table 2
The general performance of the MKT questionnaire

Number of items	Mean	SD	Range	Shapiro-Wilk Test			Cronbach's alpha
				Statistics	df	Sig.	
17	7.56	2.72	16 (0-16)	.987	206	.056	$\alpha = .610$

The detailed evidence of each item is shown in Table 3 which describes the PPSTs' responses to the SMK items in the questionnaire.

As seen in Table 3, PPSTs faced challenges in solving the tasks that focus on the various representation types in the part-whole items (Q1, Q3.2, Q3.3). Moreover, PPSTs struggled to respond to the measure sub-construct questions and struggled to explain the reason for their answers (Q2.2, Q6.1, Q6.2). However, they performed better in the measure sub-constructs, which referred to place fractions on the segment and number line (Q4.1, Q4.2, Q5). PPSTs received better scores in solving fractions problems of ratio and quotient sub-constructs than other sub-constructs, which required computation rather than explanation (Q7, Q8, Q9).

Table 3
Descriptive results of PPSTs' responses to the SMK items in the questionnaire

Items	SMK-Fractions	Max	Mean	SD	N	Percentages
Q1	CCK part-whole	1	.03	.18	7	3.4
Q2.1	CCK measure and operator	1	.35	.47	72	35
Q2.2	SCK measure and operator	1	.30	.45	61	29.6
Q3.1	CCK part-whole, operator	1	.67	.47	139	67.5
Q3.2	CCK part-whole, operator	1	.13	.33	26	12.6
Q3.3	CCK part-whole, operator	1	.16	.36	33	16
Q4.1	CCK measure	1	.74	.44	152	73.8
Q4.2	CCK measure and operator	1	.41	.49	85	41.3
Q5	CCK measure	1	.64	.48	131	63.6
Q6.1	CCK measure	1	.24	.42	49	23.8

Q6.2	CCK and SCK measure	1	.20	.40	41	19.9
Q7.1	CCK Ratio	1	.71	.45	146	70.9
Q7.2	CCK Ratio	1	.67	.47	137	66.5
Q8.1	CCK quotient	1	.74	.43	153	74.3
Q8.2	CCK quotient	1	.39	.48	80	38.8
Q9.1	CCK quotient	1	.86	.34	178	86.4
Q9.2	CCK quotient	1	.33	.47	68	33

5.2. Characteristics of PPSTs' MKT on fractions

This section describes PPSTs' performance in each sub-construct of fractions. By utilizing the result from Table 3, the items referred to each sub-construct of fractions were calculated to identify the mean scores and percentages of the correct answers. However, the operator sub-construct items, which were integrated into the other sub-constructs, were treated equally. For instance, part-whole sub-construct items (Q1, Q3.1, Q3.2, Q3.3) were calculated to determine the average scores and percentages of the items. Items Q2.1, Q2.2, Q3.1, Q3.2, Q3.3, Q4.2 were considered as the operator sub-constructs. Table 4 below presents the result of each sub-construct of fractions after calculation.

Table 4
Descriptive statistics of fractions sub-constructs

Fractions sub-constructs	Part-whole	Measure	Operator	Ratio	Quotient
Mean	.25	.41	.34	.69	.58
SD	.2	.25	.21	.45	.25
Percentages (%)	24.87	41	33.67	68.7	58.13

MKT on part-whole sub-construct

For the CCK part-whole, the majority of the PPSTs' responses in the questionnaire and interview considered the part-whole construct as the equal shape, and they were confused with the figures that divided into 4 parts as fractions. Only 3.4 percent of the participants were able to distinguish the unequal shape figures from the equal area figures. Moreover, they had insufficient knowledge to extend the different representations for improper fractions.

Regarding the SCK part-whole in item Q1, PPSTs provided an explanation of the rationale in choosing part-whole representation. However, the responses were limited based on their CCK; they explained based on the chosen answer from the questionnaire. A majority of them shaded the figures that were divided into equal shapes. Besides, PPSTs considered fractions as the figures that were divided into 4 parts as the reason for choosing fractions $\frac{1}{4}$. They chose all figures which were divided into 4 parts, both equal and unequal. For instance:

because the first figure has 4 parts too. Then, we want to take $\frac{1}{4}$ the denominator is 4, so we shade 1 part from 4 parts too. For the other figures, even though they are divided unequally, they still take 1 part among 4 parts as well. (3S211)

Question1: Which shape is divided into $\frac{1}{4}$?
Please ✓, in the box of the correct number. In cases you ✓ , color one quarter $\frac{1}{4}$.

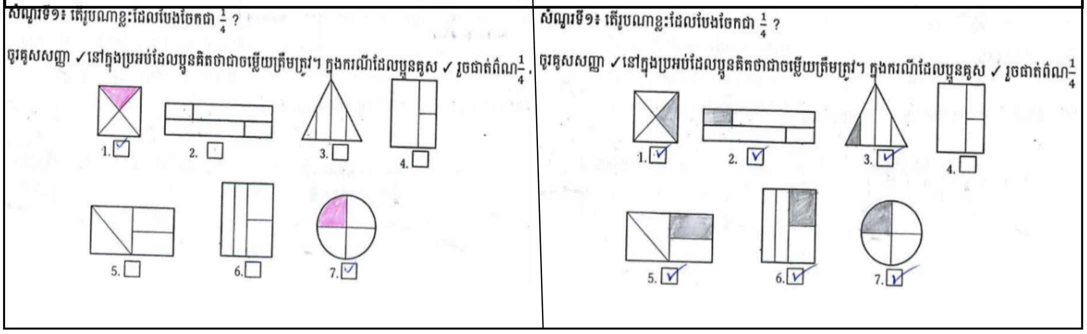


Figure 1. Example of PPSTs' responses for item Q1 in the questionnaire

Meanwhile, for KCS part-whole, PPSTs seemed to have proper knowledge in anticipating students' difficulties and misconceptions in part-whole representation which referred to the misconception regarding the figures that were divided into 4 parts, including equal and unequal parts. For instance, the following statements highlight the participants' responses:

> Students have difficulties because some figures are divided into unequal parts. Some figures have a circle that is divided equally. So, students fine it difficult to recognize which figures are fractions.(1P29)

> Students might consider the figures which are divided into 4 parts as the answers because they don't consider the unequal parts. (2P43)

For the KCT part-whole, PPSTs provided precise explanations, which considered to choose simple figures to make students easy to understand. They also expanded the examples with the unequal shape figures, which helped students recognize and distinguish the divided parts of figures. For instance, the following statements are the participants' responses:

> For teaching, I will choose numbers 1 and 7 because these figures are correct, then students will find it easy to understand. This figure is also divided into 4 parts, but it might confuse the students. So, for teaching, if we teach the confusing figures, it will make students confused as well. For figure 5, it is equal in shape and area, but for primary school students, they are hard to recognize it. (2S22)

> For teaching, I will choose the tricky figure. I will choose the correct and incorrect figures. If we use only correct figures, then we will get only the correct answer. So, we utilize some incorrect figures as well. (1P21)

MKT on measure sub-construct

In terms of the CCK measure, the results showed that PPSTs had inadequate knowledge regarding measure sub-constructs. PPSTs solved the problems correctly regarding placing fractions on the number line with 1 cm segment. However, they faced challenges in solving the problems and placing fractions on the segment, which the whole is more than one centimeter. For instance, in Figure 2 below, PPSTs found it challenging to place the improper fractions on the segment

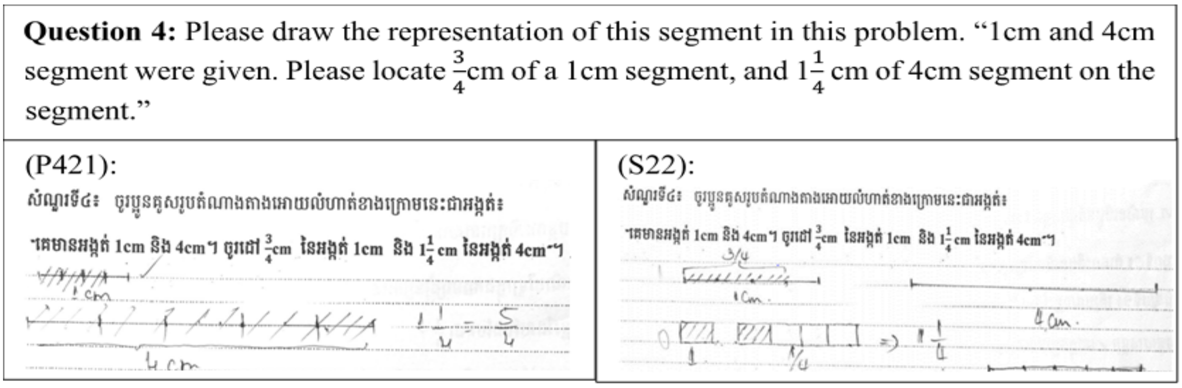


Figure 2. Example of PPSTs' marking mixed-number on the segment

For the SCK of measure sub-constructs, PPSTs' responses to the items indicated some limitations of their knowledge. PPSTs had difficulties determining fractions in which the whole is greater than 1 and solving the item Q2 utilizing part-whole sub-constructs. The part-whole sub-construct here means that they were aware that each part or taken part was $\frac{1}{3}$ of the 3 total parts of the rope, but they did not understand that this part was $\frac{1}{3}$ of 2m. Moreover, they placed fractions on the segment and the number line without understanding the specific reason and also applied decimal numbers to measure the length of the segment before placing fractions.

Regarding the KCS and KCT on measure sub-constructs in item Q2, PPSTs could anticipate students' difficulties with their limited understanding and superficial ideas. Moreover, for the KCT on measure sub-constructs, PPSTs attempted to utilize the linear model to explain the concept of the question, which focused on the $\frac{1}{3}$ *of 2 meters* to the students. However, PPSTs' responses were constrained by their SCK and KCS. The linear representation that they utilized to explain their students helped PPSTs realize their mistake in solving item Q2.

> students might get confused that 2m is the denominator, they might answer $\frac{1}{2}$ m. (2P43)

> students don't understand that the rope is 2m. They simply think the answer is $\frac{1}{3}$. But $\frac{1}{3}$ of 2m. So, students might be confused to choose $\frac{1}{3}$ m. (4P422)

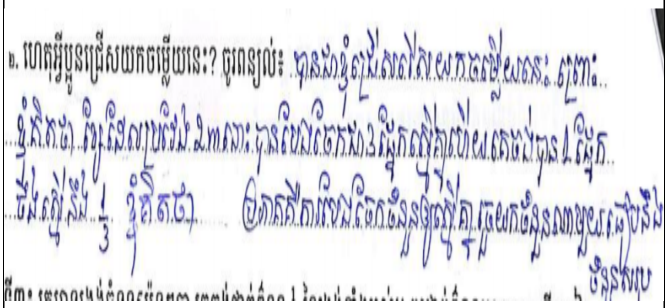
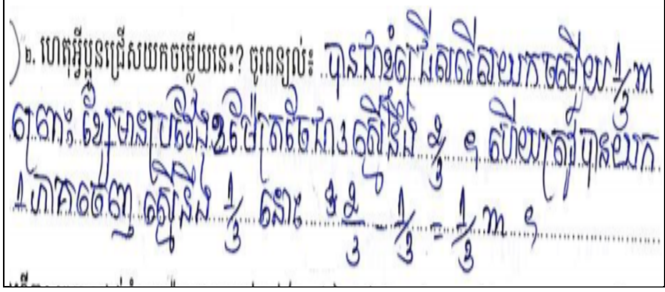
Q2.2 (take one part from 3 equal parts) 	*"because I thought the rope is 2m and is divided into 3 equal parts and she wants to take 1 part equal to $\frac{1}{3}$. I think fractions refer to dividing one whole equally, then compare the taken part with the total parts."*
Q2.2 (using subtraction operation) 	*"because the rope is 2m and is divided into 3 parts which equals to $\frac{2}{3}$m. It was taken 1 part equal to $\frac{1}{3}$. So, $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$."*

Figure 3. Examples of commonly incorrect answers in responding to Q2.2

MKT on operator sub-constructs

The operator sub-construct involves the application of the multiplication function, which is related to the calculation. The operator sub-construct in this study was indirectly investigated through the questions. The operator problems were utilized with the other sub-constructs questions that could include the operator sub-construct. For instance, in item Q2, because PPSTs were overfamiliar with the part-whole sub-construct, they did not consider multiplying the taken by the whole length of the rope. Furthermore, the operator sub-construct was less frequently mentioned in item Q10 proposed as an example for teaching fractions (see Table 5).

MKT on Ratio sub-constructs

For the CCK on ratio sub-constructs, PPSTs seemed to have proper knowledge in solving ratio problems, This was recognized as the highest performance in the questionnaire. The ratio sub-construct involves the concept of part-to-part and part-to-whole. As this study utilized a comparison of the part-to-part concept, PPSTs could write fractions or ratios of two quantities correctly. For the SCK on ratio sub-constructs, PPSTs could explain how they wrote the ratio-fractions, but they encountered challenges in comparing fractions with different denominators. To compare fractions, they employed the decimal number concept or just explained the way how they evaluated it in determining the larger amount. For the KCS on ratio sub-constructs, PPSTs emphasized the comparison fractions rather than writing ratios or fractions notation. Thus, this result showed that they had a superficial knowledge of content and students' ability. Furthermore, PPSTs could propose various instructional strategies to help students to overcome the difficulties in solving the ratio problems. However, these ideas did not correspond to the reality. They posed the ideas based on their theoretical knowledge, which was acquired from the teacher education program. They explained in words and followed the problem-solving strategy which was considered as an oral explanation. But for the "follow problem strategy", the explanation just repeated the rule in solving the problems, which involved reading the problem, noting the hypothesis, solving the problems, and concluding the answers. Thus, they had theoretical and shallow knowledge of KCT on ratio sub-constructs. The following statement illustrates this point.

> for teaching, I allow the students to solve the problem and estimate the given hypothesis. After solving it, I ask them to compare by dividing the two numbers, then they will find out which one is bigger. (1P29)

MKT on quotient sub-constructs

The result indicated that PPSTs had proper knowledge of quotient sub-constructs. They were conscious that the result of the division or sharing could be written in fractions notation. The majority of the PPSTs answered correctly in quotient sub-construct problems.. In the interviews, PPSTs could explain how they solved the Q9 problem. They were able to utilize the concept of quotient to write fractions. However, they face challenges in comparing fractions. PPSTs converted fractions to decimal numbers, and then they compared them. Moreover, they could write the fractions correctly, but they did not utilize them for comparison. They just considered sharing pizzas and estimated the amount of pizza for each person to answer the questions (see Figure 4). For the KCS on quotient sub-constructs, PPSTs focused on students' difficulties in sharing the pizza parts because of the number of pizzas and the number of people were different, which made students write fractions incorrectly. However, pre-service teachers failed to draw the representations to explain how to share the pizzas among girls and boys. Some of the participants who answered correctly also had difficulties drawing the representation, especially in sharing the pizzas for girls. This result revealed the pre-service teachers' limited knowledge of utilizing the representation model, especially for sharing objects with more than one.

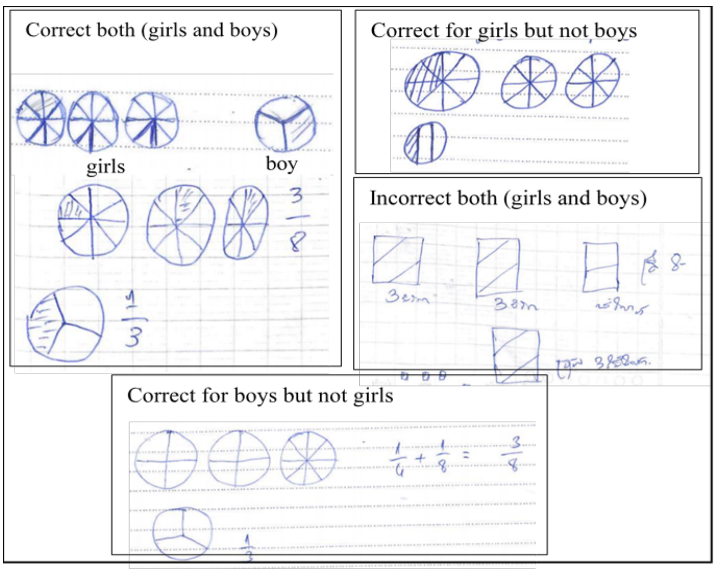


Figure 4. Common representations in item Q9

Additionally, for the KCT based on the result of item Q10 (see Table 5), PPSTs could pose examples for teaching fractions to primary school students. However, the examples of the part-whole sub-construct were mostly applied. This result revealed the participants' limited knowledge of utilizing various contexts or meanings of fractions in teaching fractions, which considered a particular meaning of fractions as part-whole representation.

Table 5
Characteristic of posed examples by PPSTs using fractions

Examples	Frequency	Percentage (%)
No answer and no meaning	63	12.7
Measure	34	6.9
Measure-subtraction	9	1.8
Measure-addition	1	0.2
Part-whole	214	43.2
Part-whole-subtraction	27	5.5
Part-whole-addition	2	0.4
Quotient	86	17.4
Quotient-subtraction	1	0.2
Operator	20	4
Ratio	8	1.6
Others (using operations of fractions)	30	6.1
Total	**495**	**100**

5.3. PPSTs' performance based on their background

The backgrounds of PPSTs were utilized for the data analysis, which referred to the teachers' educational background, training institutions pursuing in the university, math grade, and genders. Regarding the result of the Common Content Knowledge (CCK) in the questionnaire, PPSTs' performance varied based on their diverse backgrounds, excepted their genders.

Table 6
PPSTs' performance based on the teachers' education programs in each institution

	PP TEC		**SV PTTC**			
Programs	**12+2**	**12+4**	**12+2**	**F**	**df**	**Sig.**
Mean	6.54	7.61	8.43	5.28	(2, 203)	.006
SD	2.70	2.71	2.27			

Based on the result from each institution (see Table 6), PPSTs in Svay Rieng PTTC performed better than PPSTs in PPTEC, and there was a statistically significant difference between PPTEC and SVPTTC.

Meanwhile, among the 206 PPSTs, the majority of them hold grades C (24.3%), D (20.4%), and E (30.6%) in the national mathematics examination. Based on the result of their math grade, it is revealed that the PPSTs who hold high grades performed better than those with lower grades. Moreover, the result of the one-way ANOVA revealed a statistically significant difference between PPSTs' math grades and their performance of MKT test items (F (5, 200) = 7.26, $p < .001$).

By clustering the numbers of PPSTs in each institution, 54.8% of the PPSTs in SVPPTC hold higher grades (grade A, B, and C), while the score for PPSTs in PPTEC was 43.9%. For each teacher education program, the number of PPSTs who hold higher grades (grade A, B, and C) in the 12+2 program in PPTEC was 46.4%, while in the 12+4 program was 43.1%. The score for the 12+2 program in SVPPTC was 54.8%.

Table 7

PPSTs' performance based on their math grades

Math Grades	**A**	**B**	**C**	**D**	**E**	**F**
Mean	9.12	9.24	7.98	7.14	6.57	5.16
SD	2.72	2.6	2.14	2.71	2.66	1.72
N (Total 206)	16	29	50	42	63	6

As seen in Table 8, there were more PPSTs at SVPTTC who were enrolled in other universities than PPSTs at PPTEC. 69% of the PPSTs in SVPTTC are pursuing their study in other universities. For PPTEC, only 12.8% of the PPSTs are pursuing in their study at other universities. The result of Chi-square test revealed the significant difference between PPSTs who are pursuing their study in other universities and those who are not ($X^2(1, N = 206) = 57.54)$, $p < .001$).

Table 8

PPSTs' performance based on their gender, pursuing other university and institutions

		Mean	***t***	**df**	**Sig**
Gender	Male	7.96	1.55	204	$p = .122$
	Female	7.35			
PPSTs pursuing other universities	Yes	8.64	3.29	204	$p = .001$
	No	7.22			
Training institutions	PPTEC	7.34	-2.23	204	$p = .02$
	SVPPTC	8.43			

6. Discussion

The results of this study showed that the general tendency of PPSTs' MKT on fractions is quite low. The results was interpreted by each component of the MKT model and five sub-constructs of fractions. In the CCK component, PPSTs struggled to solve the part-whole sub-construct questions. This revealed the inconsistency with previous studies showing that the participants performed better and had an insight about understanding of part-whole than the other sub-constructs (Kolar, Čadež, & Vula, 2018; Lee, So & Arabeyyat, 2015; Reeder & Utley, 2017; Van Steenbrugge et al., 2014; Wright, 2008). The results about the ratio sub-construct revealed PPSTs' high performance, a finding that paralleled Van Steenburgen's (2014) study which found that the pre-service teachers understand the comparison or proposition of two quantities that the ratio concept strongly connects to the part-whole sub-constructs. However, the low performance of the part-whole sub-construct did not necessarily mean that PPSTs did not understand this subconstruct. Based on the analysis of the items and the interview data regarding part-whole sub-constructs, PPSTs considered fractions as a whole divided into equal parts without considering a whole divided into equal areas. Moreover, it is difficult for PPST to differentiate the figures which were utilized to represent fractions. They faced challenges in finding the different types of representations of fractions. This result might be caused by the limitations of utilizing representation in the primary school mathematics textbook, which emphasizes the part-whole sub-construct with a limited type of the area model. Lemon (2012) argued that the constraining of utilizing part-whole interpretation had left students with an impoverished notion of rational numbers.

Regarding the findings of the interviews, PPSTs' SCK was limited. They could solve the problems, but they found it difficult to explain the reason for calculating the fractions concept problems. This finding indicated the link between CCK and SCK. PPSTs tried to describe the reason for the procedure in solving problems with their limited CCK. As regards the SCK on measure and operator sub-constructs in item Q2, PPSTs still applied part-whole sub-constructs to solve and lack of understanding of fractions as magnitude. This result is related to the limitations of utilizing part-whole sub-constructs more than other sub-constructs in the primary school mathematics textbooks. Once again, for KCS, PPSTs understood students' difficulties and misconceptions. They were able to describe all sub-constructs of fractions. However, the explanation was insufficient and shallow based on the constraint of their SMK. If teachers teach with inadequate knowledge of fractions, they would be facing the challenges of identifying students' difficulties or misconceptions and finding the solutions or instructions to remedy students' misconceptions (Turnuklu & Yesildere, 2007). According to the KCT questions, PPSTs tended to utilize part-whole sub-constructs as the examples for teaching fractions, which focused on the area model. This result indicated that PPSTs understood the part-whole sub-constructs with the limited idea of the area representation, which led to their poor performance in the part-whole items in the questionnaire. Moreover, PPSTs could provide instructional strategies to overcome students' difficulties and eliminate misconceptions in various ways. They could explain by utilizing the representation and oral explanation. However, they faced challenges in drawing a representation of a quotient sub-construct, which has more than one object.

Furthermore, PPSTs' performance varied, depending their backgrounds, although their genders did not have any influence on their performance. The PPSTs who achieved high performance in the grade 12 national examination performed better in this survey. The Ministry of Education Youth and Sport has encouraged competent candidates to enroll in the teacher education program. However, most high-performing students in the upper secondary did not get involved in the teacher education program (Prigent, 2016; Tandon & Fukao, 2015). The background of the teacher education institution that PPSTs were enrolled in had influence on their performance as indicated by the results of the questionnaire survey. Specifically, it was found that PPSTs in PPTEC performed lower than those in SVPTTC. However, this result should be interpreted with caution because at the moment PPTEC is piloting and reforming its teacher education system and curriculum. Therefore, it might cause a misunderstanding in teaching and learning among teacher educators and pre-service teachers. The participants were second-year pre-service teachers, and they still have two years more for training. PPSTs who enrolled in the 12+2 program in PPTEC also faced difficulties in solving the fractions questions. The SVPTTC is training the 12+2 program, so PPSTs almost finish their training and achieve the curriculum learning outcome. This may have influenced their performance. Moreover, whether or not they were pursuing their study at other universities and their math grade might have impacted their performance as well. PPSTs in SVPTTC preferred to enroll in the other universities that might have enhanced their knowledge. The number of PPSTs in SVPPTC had a higher math grade than those in PPTEC.

7. Conclusion and recommendations

As the results of this study showed, PPSTs' Mathematical Knowledge for Teaching (MKT) on fractions concepts focusing on the five sub-constructs of fractions was moderately low. Their performance differed by each sub-construct of fractions and components of the MKT model. Among the five sub-constructs of fractions, the part-whole sub-construct received a poor result. They had restricted part-whole knowledge, which referred to a limited understanding of equal parts of the representations and an improper fractions representation model. Likewise, the PPSTs' SCK is considered the main issue in this study. PPSTs could solve the fractions problems (CCK); however, a large number of them could not provide an explicit explanation. It was challenging for them to describe the reason behind their answers. The result in this study revealed that PPSTs had inadequate specialized content knowledge (SCK) on fractions. Moreover, the result of PPSTs' common content knowledge (CCK) and specialized content knowledge (SCK) indicated issues regarding their PCK. The limited knowledge of SCK led PPSTs to provide a poor discussion on students' difficulties and misconceptions. The knowledge of content and student (KCS) was strongly influenced by PPSTs' SCK. Furthermore, regarding the knowledge of content and teaching (KCT), PPSTs found it difficult to provide the representation, particularly for fractions greater than 1. Moreover, in terms of finding examples to explain fractions, PPSTs tended to provide limited meanings of fractions, which mainly focus on the part-whole sub-construct.

This study's results imply that teacher education institutions play a role in improving and strengthening PPSTs' MKT on fractions, especially the SCK that revealed the strong relations the PPSTs' PCK. The part-whole sub-construct also needs to be enhanced since it was utilized as the fundamental concept of fractions in primary school education. Furthermore, it is recommended that curriculum developers expand the concept of fractions and the integration of multi-facets of fractions in both primary school education and teacher education programs. The particular concept of fractions as the part-whole sub-construct is not enough for students to understand the meaning and representations of fractions. Meanwhile, the various representation models also suggest emphasizing and providing a clear demonstration in utilizing different representation models to support pre-service primary school teachers.

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